

Calculators are not allowed
Answer the following questions.

1. Let $y = 5x^2 - 3$ and Δx be an increment of x .

(a) Find general formulas for Δy and dy . (3 points)

(b) If x changes from 3 to 3.1, then find dy (3 points)

2. (a) Find an equation of the normal line to the graph of the following equation

$$x \sin y + xy + y - \pi = 0.$$

at the point whose x -coordinate is 0. (5 points)

(b) Find the extrema of

$$f(x) = 2 \sin x - x \quad \text{on } [0, \pi]$$

($\sqrt{3} \simeq 1.73$ and $\pi \simeq 3.141$) (5 points)

3. (a) A point P moves on the circle $x^2 + (y - 2)^2 = 5$, in such a way that its y -coordinate increases at a rate of 4 units per second. How fast is its x -coordinate changing when $x = 1$? (5 points)

(b) State Rolle's theorem. Does this theorem apply to $f(x) = \sqrt{9 - x^2}$ on $[-3, 3]$? If yes, find c which satisfies the conclusion of the theorem and if not explain why not. (2+3 points)

4. Let

$$f(x) = \frac{x}{\sqrt[3]{x^2 - 1}}$$

(a) Show that $f'(x) = \frac{x^2 - 3}{3\sqrt[3]{(x^2 - 1)^4}}$ (3 points)

(b) Find the vertical and horizontal asymptotes for the graph of f (if any). (3 points)

(c) Find the intervals on which f is increasing or decreasing, and find the local extrema of f (if any). (3 points)

(d) Given that $f''(x) = \frac{2x(9 - x^2)}{9\sqrt[3]{(x^2 - 1)^7}}$, find the intervals on which the graph of f is

concave upward or concave downward, and find the points of inflection (if any) (3 points)

(e) Sketch the graph of f . (3 points)

(Good Luck)